Some Practice Problems using Mathematical Induction

1. Prove that, for $n \geq 1$, $\sum_{i=1}^{n} i^3 = \left(\sum_{i=1}^{n} i\right)^2$. (Hint: you already have a formula for $\sum_{i=1}^{n} i$.)

2. Prove that, for $n \geq 1$, $\sum_{i=1}^{n} i2^i = 2 + (n-1)2^{n+1}$.

3. Prove that if $n > 3$, then $2^n < n!$.

4. Prove that if $n > 4$, then $n^2 < 2^n$.

5. Prove that if $n > 9$, then $n^3 < 2^n$.

6. Prove that if $n \geq 1$, then $\sum_{j=1}^{n} jH_j = \dfrac{n(n+1)}{2} H_{n+1} - \dfrac{n(n+1)}{4}$.

7. Define $\{a_n\}$ by $a_0 = 0$, $a_1 = 1$, and $a_n = \dfrac{a_{n-1} + (n-1)a_{n-2}}{n}$ when $n \geq 2$. Prove that, for all $n \geq 0$, $0 \leq a_n \leq 1$.